# Study of Modal Behaviour of Viscoelastic Rotors Using Finite Element Method

A THESIS SUBMITTED IN THE PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

**Master of Technology**
**In**
**MECHANICAL ENGINEERING**

[Specialization: Machine Design and Analysis]

By

**Pavan R.Mutalikdesai**

**(211ME1345)**

**DEPARTMENT OF MECHANICAL ENGINEERING**

**NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA**

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Under the Supervision of

## Prof. H. ROY


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# CERTIFICATE

This is to certify that the thesis entitled, "***Study of Modal Behaviour of Viscoelastic Rotors Using Finite Element Method***", submitted by **Mr. Pavan R.Mutalikdesai** in partial fulfillment of the requirements for the award of MASTER OF TECHNOLOGY Degree in MECHANICAL ENGINEERING with specialization in MACHINE DESIGN AND ANALYSIS at the National Institute of Technology, Rourkela (India) is an authentic Work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date:

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**PAVAN R.MUTALIKDESAI**

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# Abstract:

*The Property which combines elasticity and viscosity is called as viscoelasticity. Such materials store energy as well as dissipate it to the thermal domain when subjected to dynamic loading. The storage and loss of energy depends upon the frequency of excitation. Modeling of elastic materials is easier, compared to the Modeling of viscoelastic materials. This work attempts to study the influence of internal material damping on the modal behaviour of a rotor shaft system. Rotary forces are generated due to the internal material damping of rotor shaft system and are proportional to the spin speed and acts tangential to the rotor orbit. These forces influence the dynamic behaviour of a rotor and tend to destabilize the rotor shaft system as spin speed increases. Hence, the modal behaviour of rotor shaft is studied by finite element method, to get better ideas about the dynamic behaviour of the rotor shaft system.*

*The dynamic characteristics of rotor systems are closely related with the rotor spin speed; hence the directivity of modes becomes very important in rotor dynamics. In the approach used in this work, a natural mode of the rotor is represented as the sum of two sub modes which are rotating to the forward and backward directions. This work explains the use of directional information when the equation of motion of a rotor is formulated in complex form. This methodology has an advantage of incorporating the directionality. Work establishes the fact that directional frequency response functions (dFRF) relate the forward and backward components of the complex displacement and excitation functions. The dFRFs are obtained as the solution to the forced vibration solution to the equation of motion.*

*The finite element analysis is used for modeling the system. The rotor shaft system with simply supported ends having three discs has been considered for the study. In this work the* *effects of internal viscous damping have been incorporated into the finite element model. In the analysis, the equations of motion are obtained to a good degree of accuracy by discretizing the rotor-shaft continuum using 2-noded finite Rayleigh beam elements. These equations are used for eigenanalysis. A finite element code is written in MATLAB to find out the eigenvalues, eigenvectors and modal damping factor. Stability limit of spin speed and effect of modal damping factor on the spin speed have been studied. This work establishes the fact that directional frequency response functions (dFRF) relate the forward and backward components of the complex displacement and excitation functions. The dFRFs are obtained as the solution to the forced vibration solution to the equation of motion. The Campbell diagram and the dFRF plots are obtained by using the Matlab.*

***Keywords:*** *Internal Material Damping, Modal analysis, Modal Damping Factor, Stability Limit of Spin Speed, directional Frequency Response Function (dFRF).*

# CHAPTER ONE

# Introduction

## 1.1. BACKGROUND AND IMPORTANCE:

**Rotordynamics** is a specialized branch of applied mechanics concerned with the behavior and diagnosis of rotating structures. It deals with behavior of rotating machines ranging from very large systems like power plant rotors, for example, a turbo generator, to very small systems like a tiny dentist’s drill, with a variety of rotors such as pumps, compressors, steam turbines, motors, turbo pumps etc. as used for example in process industry. The principal components of a rotor-dynamic system are the shaft or rotor with disk and the bearing. The shaft or rotor is the rotating component of the system. Additionally, machines are operated at high rotor speeds in order to maximize the power output. At its most basic level rotordynamics is concerned with one or more mechanical structures (rotors) supported by bearings and influenced by internal phenomena that rotate around a single axis. The supporting structure is called a stator. Rotors are the main sources of vibration in most of the machines. At higher speeds of rotation, the vibrations caused by the mass imbalance results in some serious problems (Rao [2]). So it becomes necessary to limit the vibrations for operational safety and stability. It can be done by proper assessment of dynamics of system. The dynamic behavior of a mechanical system must be examined in its design phase so that you can determine whether it will present a satisfactory performance or not in its condition of the planned operation. The natural frequencies, damping factors and vibration modes of these systems can be determined analytically, numerically or experimentally.

Unlike the viscoelastic structures (which do not spin) viscoelastic rotors are acted upon by rotating damping force generated by the internal material damping, that tends to destabilize the rotor shaft system by generating a tangential force proportional to the rotor spin speed. Thus a reliable model is necessary to represent the constitutive relationship of a rotor-material by taking into account the internal material damping for understanding the dynamic behaviour of a viscoelastic rotor. Such a model is useful for getting an idea about safe speed ranges of rotation, where the rotor is stable.In this work the effects of internal viscous damping have been incorporated into the finite element model.

The study for rotary machines, however, requires a more careful and detailed analysis, because the rotation movement of the rotor significantly influences the dynamic comportment of the system, making the modal parameters dependent on the rotation of the machine (Lee [3]). The gyroscopic effect couples the rotation movement and, as it's known, it is dependent on the rotation speed of the rotor. Therefore, it is expected that the natural frequencies and vibration modes of a rotating machine also depend on the system speed. Thus, unlike non rotating structures, rotors have two different kinds of modes, known as forward and backward modes due to the rotor spin.

In any individual Frequency Response Function (FRF) plot of a rotor the negative frequency region of the FRF is merely a duplicate of the positive frequency region. Therefore, it is only necessary to deal with one region of the FRF, conventionally the positive one (Mesquita et al. [4]). By the use of traditional modal analysis, it becomes difficult to identify the directivity of a mode, forward or backward. To overcome this problem, a new method called complex modal analysis for the rotors has been introduced in this work. The equation of motion is formulated by using complex variables representing displacement and excitation. The dFRFs (Directional Frequency Response Function) are obtained as the solution to the forced vibration solution to the equation of motion. The advantage of this methodology is that the directionality can be incorporated, which was not possible by using the traditional modal analysis. The method separates the backward and forward modes in the dFRF, so that effective modal parameter identification is possible. The FRF obtained by the method is discussed with special attention to its implied directional information. It is shown that the FRFs obtained as such relate forward and backward responses to forward and backward excitations, and become the same functions that were defined as the directional FRFs by (Lee [3]).

## 1.2. VISCOELASTICITY:

Viscoelasticity is a property that combines elasticity and viscosity. These materials store the energy as well as dissipate it under dynamic deformation. Thus, the stress in such materials is not in phase with the strain. Due to these properties, it is extensively used in various engineering applications for controlling the amplitude of resonant vibrations and modifying wave attenuation and increasing structural life through reduction in structural fatigue (Dutt and Nakra [5]).

Williams [6] did the Structural Analysis of Viscoelastic Materials. The classical theory of elasticity states that for sufficiently small strains, the stress in an elastic solid is proportional to the instantaneous strain and is independent of the strain rate. In a viscous fluid, according to the theory of hydrodynamics, the stress is proportional to the instantaneous strain rate and is independent of the strain (Williams [6]). Viscoelastic materials exhibit both solid and fluid behavior. Examples of Such materials are plastics, amorphous polymers, glasses, ceramics, and biomaterials (muscle). Viscoelastic materials are characterized by constant-stress creep and constant-strain relaxation. The Stress-Strain Curves for a purely elastic and a viscoelastic material are shown in figure 1.1. There’s a loss of energy during loading and unloading time. Because of this reason, the stress strain curve for viscoelastic material is elliptic in nature. The area enclosed by the ellipse is a hysteresis loop and shows the amount of energy lost (as heat) in a loading and unloading cycle.

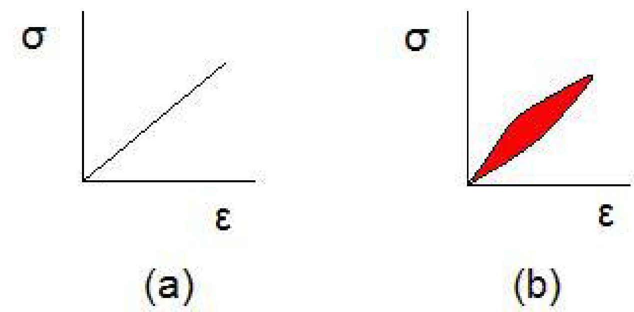



**Figure 1.1 Stress Strain Curve**

Some phenomena in viscoelastic materials are:

[1] If the stress is held constant, the strain increases with time (creep).

[2] If the strain is held constant, the stress decreases with time (relaxation).

[3] The effective stiffness depends on the rate of application of the load.

[4] If cyclic loading is applied, hysteresis (a phase lag) occurs, leading to dissipation of mechanical energy.

[5] Acoustic waves experience attenuation.

[6] Rebound of an object following an impact is less than 100%.

[7] During rolling, frictional resistance occurs.

## 1.3 MODAL ANALYSIS:

Modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes. It's used to formulate a mathematical model for its dynamic behavior. The formulated mathematical model is known as the modal model of the system and the information for the characteristics is known as its modal data. Modal analysis has evolved into a standard tool for structural dynamics problem analysis and design optimization (Chouksey et al. [11]]).

Modes are inherent properties of a structure. These are determined by the material properties (mass, damping, and stiffness), and boundary conditions of the structure. Each mode is defined by a natural (modal or resonant) frequency, modal damping, and a mode shape.

### 1.3.1. VARIOUS METHODS OF MODAL ANALYSIS

Modal analysis is based upon the fact that the vibration response of a linear time-invariant dynamic system can be expressed as the linear combination of a set of simple harmonic motions called the natural modes of vibration (He and Fu [7]). The natural modes of vibration are inherent to a dynamic system and are determined completely by its physical properties (mass, stiffness, damping). Each mode is described in terms of its modal parameters: natural frequency, the modal damping factor and mode shape. The mode shape may be real or complex. The degree of contribution of each natural mode in the overall vibration is determined both by properties of the excitation sourceand by the mode shapes of the system. Modal analysis involves both theoretical and experimental techniques. (He and Fu [7]) gave details about the various methods of modal analysis.

**THEORETICAL MODAL ANALYSIS**:

Physical model of a dynamic system comprising its mass, stiffness and damping properties are used to study the dynamic behaviour of the system. Procedure involves finding of mass, stiffness and damping properties of the system. Then the equation of motion of the system is found out.

The solution of the equation of motion provides the natural frequencies and mode shapes of the system considered and its forced vibration responses. More realistic physical model consists of mass, stiffness and damping properties in terms of their mass, stiffness and damping matrices. These matrices are incorporated into the equation of motion of the system. The superposition principle of a linear dynamic system helps us to transform these equations into a typical eigenvalue problem. The solution of the eigenvalue problem provides the modal data of the system. Finite element analysis empowers the discretization of almost any linear dynamic structure and hence has greatly enhanced the capacity and scope of theoretical modal analysis.

**EXPERIMENTAL MODAL ANALYSIS:**

Rotating machines appear in almost every aspect of our modern life for example cars, aero-planes and steam-turbines.These all have many rotating structures whose dynamics need to be modeled, analysed and improved. The stability and the response levels of these machines, predicted by analytical models, must be validated experimentally. For this reason modal testing needs to be performed. The practice of modal testing involves measuring the FRFs or impulse responses of a structure. The FRF measurement can be done by using the response recorded by the accelerometer and the data acquisition system. For small structures modal hammer can be used for the excitation and for large structures exciter is used for applying the force.

Experimental modal analysis involves three steps: test preparation, frequency response measurements and modal parameter identification.

1. Test preparation involves selection of a structure's support, type of excitation force(s), location(s) of excitation, data acquisition system to measure force(s). Structure is divided in to number of parts. Accelerometers are connected at the selected nodes.
2. Then the impact force is applied at some locations using the exciter and corresponding response are noted using the data acquisition system. Using the responses and applied force, the FRF matrix is obtained, which is then analysed to identify modal parameters of the tested structure.

Modal analysis concepts have not been applied to rotor systems as extensively as they have to other structural dynamic systems. Rotor systems force the reconsideration of some of the basic assumptions applied in modal analysis of other structures. Jeffcott [1] provided a very basic model of a rotor. Initial assumptions made by him are,: (i) No damping is associated with the rotor, (ii) Axially Symmetric rotor, and (iii) The rotor carries a point mass. Later, the model was expanded to take care of damping. Jeffcott rotor model is an oversimplification of real-world rotors and retains some basic characteristics. It allows us to gain a qualitative insight into important phenomena typical of rotor dynamics, while being much simpler than more realistic models.

Between the work of Jeffcott [1] and the start of World War II there was much work in the area of instabilities and modeling techniques culminating in the work of Prohl and Myklestad which led to the Transfer Matrix Method (TMM) for analyzing rotors. Using this TMM technique researchers found some difficulties by the peculiar approach developed for the study of the attitude dynamics of spinning spacecraft. The wide diffusion of the finite element method (FEM) deeply influenced also the field of rotor dynamics Strictly speaking; usual general purpose FEM codes cannot be used for rotor dynamic analysis owing to the lack of consideration of gyroscopic effects. It is true that a gyroscopic matrix can be forced in the conventional formulation and that several manufacturers use commercial FEM codes to perform rotor dynamic analysis, but the rotor dynamic field is one of these applications in which purposely written, specialized FEM codes can give their best. Through FEM modeling, it is possible to study the dynamic behavior of machines containing high-speed rotors in greater detail and consequently to obtain quantitative predictions with an unprecedented degree of accuracy. Nelson and McVaugh [8] written extensively history of rotor dynamics and most of the work is based on Finite Element Methods.

In particular, rotor systems do not in general obey Maxwell's reciprocity theorem; system matrices are nonsymmetric. System matrices also depend upon rotor speed due to the presence of gyroscopic effects which lead to skew-symmetry in the damping matrix; this is a linear function of rotor speed. Also, support characteristics are nonsymmetric for commonlyused bearing types and they vary widely with rotor speed. This work is aimed primarily to study the dynamic behavior of fixed-base heavy rotors used in most of the power plant industries. Many Researchers worked in the field of Rotordynamics. Some contributions are explained here.

Dutt and Nakra [5] carried out stability analysis of a rotor systems in which , rotor disc is placed in the middle of a massless shaft, having linear elasticity and internal damping and supported on Viscoelastic supports. They obtained results for cases with purely elastic or viscously damped and flexible supports. They found that suitably chosen Viscoelastic supports can increase the stability zones of the system considerably compare to the other type of the supports. Nelson and McVaugh [8] presented a procedure for dynamic modeling of rotor bearing systems which consisted of rigid disks, distributed parameter finite rotor elements, and discrete bearings. They presented their formulation in both a fixed and rotating frame of reference. They developed a finite element model including the effects of rotary inertia, gyroscopic moments, and axial load. They utilized a coordinate reduction method to model elements with variable cross-section properties. Their model included linear stiffness and viscous damping cases only. The study included an overhung rotor with two sets of bearing parameters: (i) undamped isotropic, (ii) undamped orthotropic. The comparison of results was made with an independent lumped mass analysis. Zorzi and Nelson [9] provided a finite element model for a multi-disc rotor bearing system. The model was based on Euler-Bernoulli beam theory. Their work included viscous as well as hysteresis damping forms of material damping (internal damping) of the system. They demonstrated that the material damping in the rotor shaft introduces rotary dissipative forces which are tangential to the rotor orbit, well known to cause instability after certain spin speed. Both forms of internal damping destabilize the rotor system and induce non-synchronous forward precession. They also demonstrated the effects of anisotropic bearing stiffness and external damping.

Ku [10] included the combined effects of transverse shear deformations and the internal viscous and hysteretic damping in their analysis. Results of forward and backward whirl speeds and damped stability are presented and compared with other previously published works. Better convergence and high accuracy of the present finite element model are demonstrated with numerical examples.

Chouksey et al. [11] studied the influences of internal rotor material damping and the fluid film forces (generated as a result of hydrodynamic action in journal bearings) on the modal behaviour of a flexible rotor-shaft system. Due to journal bearing and the internal material damping introduce tangential forces increasing with the rotor spin speed and tend to destabilize the system. Under this system of forces the modal behaviour of the rotor-shaft is studied to get better ideas about the dynamic behaviour of the system. The stability limit speed is calculated and the effect of modal damping factor on the spin speed is studied.The mode shapes of the rotor are also found out.Laszlo [12] studied the stability analysis of self-excited vibrations of linear symmetrical rotor bearing systems with internal damping using the finite element method. The rotor system consists of uniform circular Rayleigh shafts with internal viscous damping, symmetric rigid disks, and discrete isotropic damped bearings. The effect of rotary inertia and gyroscopic moment are also included in the mathematical model. By doing analysis it’s proved that the whirling motion of the rotor system becomes unstable at all speeds beyond the threshold speed of instability. It's found that the rotor stability is improved by increasing the damping provided by the bearings, whereas increasing internal damping may reduce the stability threshold.

Jei and Lee [13] studied the analysis of asymmetrical rotor-bearing systems. They considered the effect of rotary inertia, gyroscope, transverse shear deformation, internal damping and gravity during finite element modeling. , the modal transform technique is applied to reduce the size of the final resulting matrices. Finally the accuracy of the finite element model and the modal transform technique is demonstrated. Agostini and Capello [14] worked on the vibration analysis of vertical rotors by considering the gravitational and gyroscopic effects. Two modes namely forward and backward are obtained separately by using complex modal analysis in conjunction with the finite elements method in Matlab. Numerical simulations have been satisfactory when compared with the existing literature.

Modal analysis is a useful tool to get an idea about the dynamic behavior of a system and hence is considered to be important for dynamic design. Modal analysis of rotors differs from modal analysis of structures due to additional forces induced by rotor spin like gyroscopic, tangential and rotating damping forces. These forces change the nature of system matrices and make them asymmetric and speed dependent. Therefore all modal characteristics of rotors are closely related to rotor spin speed.

By doing the modal analysis we find out the resonant frequencies and the modal damping factors. It's necessary to calculate these factors because of following reasons.

### 1.3.2. INTERNAL DAMPING

All types of damping associated to the non-rotating parts of the structure have a stabilizing effect on the system. On the other hand damping associated to rotating parts can result in instability in supercritical ranges. Due to the Rotation of rotors rotary damping forces arise , which increase with the spin speed and acts tangential to the rotor orbit. These systems of forces result in the instability of rotor-shaft systems. Thus a reliable model is necessary to represent the rotor internal damping for correct prediction of stability limit of spin speed (SLS) of a rotor-shaft system. Rotary machines, like motors, compressors and turbines are very common and widely used. Now a days the designers of these machines have been required to meet very severe specifications from the demands of high speed operating power or improvements in efficiency and reliability for the design. In order to meet such requirements, we have to find some robust and reliable mathematical models, along with special numerical solution procedures, which enable designers to make an accurate assessment of the relevant parameters, the critical speeds and the dynamic behavior of the system. Response of the system to an unbalance excitation is considered to be of much importance in order to design for increased speeds of rotation, to optimize weight and to improve reliability.

### 1.3.3. RESONANT SPEED

Many of the structures can be under resonance, i.e. to vibrate with excessive oscillatory motion. Resonant vibrations arise due to the interaction between the inertial and elastic properties of the materials within a structure. Resonance is often the cause for many of the vibration and noise related problems that occur in structures and operating machinery. For the better understanding of any structural vibration problem, its necessary to identify the resonant frequencies of a structure. Today, modal analysis has become a widespread means of finding the modes of vibration of a machine or structure . In the development of a new product, structural dynamics testing is used to assess its real dynamic behavior.

### 1.3.4. MODAL DAMPING FACTOR

In order to understand the importance of calculating the modal damping factor of a system, consider an example of aeroplanes. Wings of airplanes can be subjected to similar flutter phenomena during flight. Before the release of an airplane, flight flutter tests have to be performed to detect possible flutters. The classical flight flutter testing approach is to expand the flight envelope of an airplane by performing a vibration test at constant flight conditions. Curve fitting method is used to estimate the resonance frequencies and damping ratios. Then these are plotted against the flight speed. The damping values are then used to determine whether it is safe to proceed to the next flight test point. When one of the damping values tends to become negative, fluttering starts. Ground vibration tests as well as numerical simulations and wind tunnel tests are done before starting the flight tests, to get some prior insight into the problem. Hence by the modal analysis we find out the modal damping factors. These are plotted against the spin speed. From this graph we can find out the stability of the system.

### 1.3.5. DIRECTIONAL FREQUENCY RESPONSE FUNCTIONS (DFRF):

The concepts of traditional modal analysis in stationary structures have been applied in the analysis of rotating structures. However, the analysis for rotating structures requires a more general theoretical development. Due to the rotation, gyroscopic effects appear resulting in non-symmetric matrices in the equations of motion of the system and, as consequence the FRF (Frequency Response Function) matrix does not obey the Maxwell’s reciprocity theorem (Lee [3]).

Rotors, unlike other stationary structures, exhibit a particular phenomenon in their modal characteristics (Laszlo Forrai [12]). As rotor starts rotating, it gives raise to two different kinds of modes known as the backward and forward modes. Although the presence of backward and forward modes in rotor dynamics has been extensively investigated in the literature, the directivity of the modes is often neglected in the usual formulations in the dynamic analysis of rotors. If we neglect the important modal parameters such as the modal vectors and the adjoint modal vectors, it becomes difficult to find out the mode shapes.

In any individual FRF plot of a rotor, the negative frequency region and positive frequency region are merely duplicate of each other. Because of this reason it is only necessary to deal with positive frequency region of FRF, because it yields some physical meaning. By using the traditional modal analysis, the directivity of a mode whether forward or backward, cannot be easily distinguishable in the frequency domain(Mesquita et al. [4]). The complex modal analysis is nothing but, the application of classical modal analysis principles to rotating systems, where complex variables are used to represent input and output of the system. This methodology has the ability of incorporating directionality, which was not possible using the traditional modal analysis(Kessler and Kim [15]).This method separates the backward and forward modes in the dFRF (Directional Frequency Response Function), so that effective modal parameter identification is possible. The frequency response function (FRF) obtained by this method is discussed with special attention to its implied directional information. It is shown that the FRFs obtained, relate forward and backward responses to forward and backward excitations, and become the same functions that were defined as the directional FRFs (dFRFs) by Lee [3]. Complex modal testing theory developed in this project gives not only the directivity of the backward and forward modes ,but also completely separates those modes in the frequency domain so that effective modal parameter identification is possible. The concept of complex modal analysis was first proposed by Lee[3].

Kessler and Kim [16] used a complex variable description of planar motion. It incorporates directivity as inherent Information. They used the directional information explicitly by formulating the equation of motion of a rotor in complex variables. Lee [3] proposed a new modal testing method, termed complex modal testing, for the modal parameter identification of rotating machinery and compared with the classical method. Complex modal testing allows clear physical insight into the backward and forward modes. This also enables the separation of those modes in the frequency domain so that effective modal parameter identification is possible.

Jei and Kim [17] proposed a new modal testing theory to separate the rotor vibration into positive and negative frequency regions. The amplitude and directivity variations of frequency response functions in positive and negative frequency regions are discussed using complex modal displacement. A method to identify the directivity of modes such as forward and backward is suggested using the frequency response function obtained by the proposed modal testing theory.

Directional frequency response functions (dFRFs) provide modal directivity and separation in the frequency domain. The work establishes the fact that dFRFs relate the forward and backward components of the complex displacement and excitation functions.

## 1.4 MOTIVATION OF THE WORK:

Because of greater demands made on improving the performance of rotating machinery the influence of rotordynamics on the day to day safe operation and long term health of the equipment is becoming important. Increased power output through the use of higher-speed more flexible, rotors has increased the need, at all stages of design. For the optimum and safe design of these components, understanding of the interaction of the resulting static and dynamic forces between the rotating element and stationary components becomes important.

The acceptable performance of a turbo machine depends on the adequate design and operation of the bearing supporting a rotor. Turbo machines also include a number of other mechanical elements which provide stiffness and damping characteristics and affect the dynamics of the rotor-bearing system.

The adequate operation of a turbo machine is defined by its ability to tolerate normal (and even abnormal) vibrations levels without affecting significantly its overall performance (reliability and efficiency). The rotordynamics of turbo machinery encompasses the structural analysis of rotors (shafts and disks) and design of bearings that determine the best dynamic performance given the required operating conditions. The best performance is ensured when the natural frequencies (and critical speeds) with amplitudes of synchronous dynamic response are within required standards and the absence of sub synchronous vibration instabilities. A rotordynamics analysis considers the interaction between the elastic and inertia properties of the rotor and the mechanical impedances from the bearing supports.

The most commonly occurring problems in rotordynamics are Sub harmonic rotor instabilities.

Method to avoid Sub harmonic rotor instabilities may be avoided by:

1. Increasing the natural frequency of rotor system.
2. Eliminating the instability.
3. Introducing damping to increase the speed above the operating speed range.

Rotor dynamic instabilities have become more common as the speed and power of turbo machinery increased. These instabilities can sometimes be erratic, resulting in increasing vibration amplitudes for no apparent reason. So it becomes necessary to limit the vibrations for operational safety and stability. It can be done by proper assessment of dynamics of system. The purpose of modal analysis is to get an idea about the dynamic behaviour of the system. Using this analysis we can find out the critical and stability limit speeds and from this we can minimize the vibrations. Hence the modal analysis can be used as a tool to get an idea about the dynamic behaviour of the system.

## 1.5 LAYOUT OF THE PRESENT WORK:

In this work, Finite element method is used for the analysis. Here the rotor–shaft system is modeled by considering the Euler-Bernoulli beam theory and discretised by finite element method to derive equations of motion (Rao [2], Zorzi and Nelson [9]). An example of three discs rotor system is presented here. For the purpose of numerical analysis of the system, rotor shaft system with the simply supported ends has been considered. Following Zorzi and Nelson [9], the constitutive relationship is written, where Voigt model (2-element spring dashpot model) is used to represent the rotor internal damping. A finite element code is written using MATLAB to find out the eigenvalues and eigenvectors. Eigenvalues are calculated and the imaginary parts, which show the natural frequencies of the system, are used to plot the Campbell diagram. Decay rate is plotted by using the maximum value of the real part of all eigenvalues with the spin speed. Eigen values are also used to study the effect of modal damping factor on the spin speed. From these plots, the stability of the system can be studied and the stability limit of spin speed (SLS) can be found out. Using the eigenvectors the mode shapes of the rotor are found out. Further this code is extended to get the dFRF plot, from which the directivity of the modes can be found out. Based on that review, the objective and scope proposed in this work are as follows.

1. Finding out the equations of motion and Development Finite Element formulation of the viscoelastic rotor. Finite element method is used to discritize the rotor continuum.

2. Dynamic behaviour of a mild steel rotor is predicted through viscoelastic modeling of the continuum to take into account the effect of internal material damping. Stability limit of spin speed, effect of modal damping factor on the spin speed are studied. The mode shapes are found out using the eigenvectors. Further dFRF plot is obtained by using the eigenvectors in Mat Lab.

# CHAPTER TWO

## Viscoelastic Rotor and Its Modeling

This chapter forms the basis of the entire work as it presents the derivation of the equations of motion of a viscoelastic rotor and Development Finite Element analysis procedure. The equation of motion is further used to obtain the eigenvalue and eigenvector for various kind of modal analysis.

Modal analysis is a useful tool to get an idea about the dynamic behaviour of a system and hence is considered to be important for dynamic design. One of the characteristics of rotating machinery is that they generally do not abide by the principle of reciprocity and are referred to as 'Non Self Adjoint Systems' (NSA). This property results in two sets of eigenvectors, referred to as the 'left hand' and 'right hand' eigenvectors. The right hand eigenvectors are those generally associated with the mode shapes, while the left hand eigenvectors are the set, which are obtained by analysing the transformed equations and indicate the pattern of forces associated with a single mode.

## 2.1 EQUATION OF MOTION:

In this section the mathematical modeling of viscoelastic rotor shaft is represented in time domain. The finite element model of the viscoelastic rotor shaft system is based on the Euler-Bernoulli beam theory. The equation of motion is obtained from the constitutive relation where the damped shaft element is assumed to behave as Voigt model i.e., combination of a spring and dashpot in parallel.

Figure 2.1 shows the displaced position of the shaft cross section. $(v, w)$ Indicate the displacement of the shaft Centre along Y and Z direction and an element of differential radial thickness $dr$ at a distance $r$ (where $r$ varies from 0 to $r_0$) subtending an angle $d(\Omega t)$ where $\Omega$ is the spin speed in rad/sec and $\Omega t$ varies from 0 to $2\pi$ at any instant of time $'t'$. Due to transverse vibration the shaft is under two types of rotation simultaneously, i.e., spin and whirl. $\omega$ is the whirl speed.

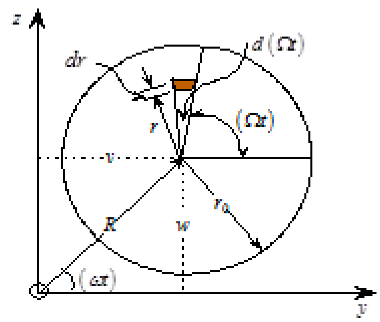



**Figure 2.1 Displaced Position of the Shaft Cross Section**

The dynamic longitudinal stress and strain induced in the infinitesimal area are $\sigma_x$ and $\varepsilon_x$ respectively. The expression of $\sigma_x$ and $\varepsilon_x$ at an instant of time are given as Zorzi and Nelson [9].

$$\sigma_x = E\left(\varepsilon + \eta_v \dot{\varepsilon}\right);\ \varepsilon_x = -r\cos\left[\left(\Omega - \omega\right)t\right]\frac{\partial^2 R(x,t)}{\partial x^2} \tag{1}$$

Where, $E$ is the Young's modulus, $\eta_v$ is viscous damping coefficient.

Following Zorzi andNelson [9] the bending moments at any instant of time about the $y$ and $z$-axes are expressed as:

$$M_{zz} = \int_0^{2\pi}\int_0^{r_0} -\left(v + r\cos\left(\Omega t\right)\right)\sigma_x r\,dr\,d(\Omega t) \tag{2}$$

$$M_{yy} = \int_0^{2\pi}\int_0^{r_0} \left(w + r\sin\left(\Omega t\right)\right)\sigma_x r\,dr\,d(\Omega t)$$

After utilizing equation (1) in equation (2) and following Zorzi and Nelson [9], the governing differential equation for one shaft element is given as:

$$\left(\left[M_T\right]+\left[M_R\right]\right)\left\{\ddot{q}\right\}+\left(\eta_v\left[K_B\right]-\Omega[G]\right)\left\{\dot{q}\right\}+\left(\left[K_B\right]+\eta_v\Omega\left[K_C\right]\right)\left\{q\right\}=\{B\} \tag{3}$$

In the preceding equation $\left[M_T\right]_{(8\times8)}$, $\left[M_R\right]_{(8\times8)}$, $[G]_{(8\times8)}$, $\left[K_B\right]_{(8\times8)}$ and $\left[K_C\right]_{(8\times8)}$ are the translational mass matrix, rotary inertia matrix, gyroscopic matrix, bending stiffness matrix and skew symmetric circulatory matrix, respectively. The expressions for those matrices are given below.

$$\left[M_T\right]=\int_0^l \rho A\phi(x)\phi(x)^T\,dx,$$

$$\left[M_R\right]=\int_0^l \rho I\phi'(x)\phi'(x)^T\,dx, \tag{3a}$$

$$[G]=\int_0^l 2\rho I\phi'(x)\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\phi'(x)^T\,dx,$$

$$\left[K_B\right]=\int_0^l EI\left[\phi''(x)\right]\left[\phi''(x)\right]^T\,dx,$$

$$\left[K_C\right]=\int_0^l EI\left[\phi''(x)\right]\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\left[\phi''(x)\right]^T\,dx$$

Where, $\rho$ is the mass density, '$I$' is the area moment of inertia, $\left(I=\int_A y^2 dA\right)$.

The Hermits shape function matrix, $\phi(x)$, is given by $\left[\phi(x)\right]=\begin{bmatrix}\{\phi_{xy}(x)\} & \{0\} \\ \{0\} & \{\phi_{zx}(x)\}\end{bmatrix}$,

Where subscripts in the elements show the respective planes

The equation of motion for whole system is obtained by assembling the element matrix to global matrix and it is rewritten as:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{B\} \tag{4}$$

Where $[M]$, $[C]$ and $[K]$ are the global mass, damping and stiffness matrices, respectively and $\{B\}$ is the external force applied. Their expressions are written as:

$$[M]=[M_T+M_R];[C]=\eta_v[K_B]-\Omega[G];[K]=[K_B]+\eta_v\Omega[K_C]$$

The disc mass is incorporated with the global mass matrix at appropriate node. The global damping matrix contains the gyroscopic effects of shaft and disc, and effects of rotating and non rotating damping.

Equation (4) once again is appended by an identity equation to constitute the states space equation.

$$[A]\{\dot{X}\}+[B]\{X\}=\{P\} \tag{5}$$

Where,

$$[A]=\begin{bmatrix}C & M \\ M & 0\end{bmatrix}, \qquad [B]=\begin{bmatrix}K & 0 \\ 0 & -M\end{bmatrix}, \{X\}=\begin{Bmatrix}\{0\} \\ \{q\}\end{Bmatrix}, \qquad \{P\}=\begin{Bmatrix}\{0\} \\ \{B\}\end{Bmatrix} \tag{6}$$

Free vibration Equation of equation (5) is an eigenvalue problem and can be written by assuming, $\{u\}=e^{\lambda t}\{y\}$.

$$\lambda[A]\{X\}+[B]\{X\}=\{0\} \tag{7}$$

Where, $\lambda$ is system's complex eigenvalue, for which the stability can be predicted from the real part and the imaginary part indicates the natural frequency.

## 2.2FREE VIBRATIONAL ANALYSIS:

Free vibration analysis of the system of equations in (4), gives eigenvalues $\lambda_r$, right eigenvector $u_r$ and left eigenvector $v_r$ in real space and are connected by equations given in (8); the subscript 'r' denotes mode number.

$$(\lambda_r{}^2M+\lambda_rC+K)u_r=0 \text{ and } (\lambda_r{}^2M^T+\lambda_rC^T+K^T)v_r=0 \tag{8}$$

Where$r$=1to 4($N$+1)

Where $N$ is the number of elements

The r$^{th}$ eigenvalue problem associated with equation (5) may be written after following Lee [3] as:

$$\lambda_rA\Psi_r=B\Psi_r \text{ and } \lambda_rA^T\Phi_r=B^T\Phi_r \tag{9}$$

Where $r$=1 to 8($N$+1), $\lambda_r$ is r$_{th}$ eigenvalue, $\Psi_r$ and $\Phi_r$ stand for the r$^{th}$ right hand and left hand eigenvectors, respectively in state space and are given by –

$$\Psi_r=\begin{Bmatrix}\lambda_ru_r\\ u_r\end{Bmatrix},\Phi_r=\begin{Bmatrix}\lambda_rv_r\\ v_r\end{Bmatrix} \tag{10}$$

$\Psi_r$ And $\Phi_r$ may be biorthonormalized so as to satisfy

$$\Phi_i^TA\Psi_r=\delta_{ir}:\Phi_i^TB\Psi_r=\lambda_r\delta_{ir} \tag{11}$$

Where$i$,$r$ = 1 to 8($N$+1), and $\delta_{ir}$,stands for the kronecker delta(i.e.= 1, when $i$ = $r$ and = 0, when $i\neq r$)

For $r$ = 1 to 8($N$+1), $\psi_r$ $\varphi_r$ and $\lambda_r$ may be written in compactform as $\Psi$ **,** $\Phi$ and $\Lambda$ respectively and are given by

$$\Psi = \left[\psi_1\, \psi_2 \ldots \psi_{8(N+1)}\right],\ \Phi = \left[\varphi_1\, \varphi_2 \ldots \varphi_{8(N+1)}\right],,\ \Lambda = diag\left[\lambda_1\ \lambda_2 \ldots \lambda_{8(N+1)}\right] \tag{12}$$

$$\Phi^T A \Psi = I \quad ; \quad \Phi^T B \Psi = \Lambda \tag{13}$$

In equation (13) the symbol, '$I$' represents identity matrix of size 8($N$+1) × 8($N$+1).

## 2.3 DEVELOPMENT OF FREQUENCY RESPONSE FUNCTION MATRIX:

Assuming a solution of $X(t)$, the state vector in equation (5), as a linear combination of the right hand eigenvectors $\psi_r$ multiplied by modal co-ordinates $\xi_r(t)$ ,as given in equation (14):

$$X(t) = \sum_{r=1}^{8(N+1)} \Psi_r \xi_r(t) = \Psi \xi(t) \tag{14}$$

Substituting it into equation (5), pre multiplying throughout by $\Phi^T$ and incorporating the bio-orthogonality conditions, the equation (15) is obtained.

$$\Phi^T A \Psi \dot{\xi}(t) = \Phi^T B \Psi \xi(t) + \Phi^T F(t) \tag{15}$$

Using orthonormality relations of equation (13), the equation (15) may be transformed, as shown below, into equation (16).

$$\dot{\xi}(t) = \Lambda \xi(t) + n(t) \tag{16}$$

Where, n(t)=$\Phi^T$F(t) ,represents the modal excitation vector.

Equation (16) represents a set of independent modal equations, and may be written as in equation (17):

$$\dot{\xi}_r(t) = \lambda_r \xi_r(t) + n_r(t) \tag{17}$$

$r$=1 to 8($N$+1)

Where, $n_r(t) = \Phi_r{}^T F(t)$, is the generalized time varying forcing function exciting the r[th] mode.

Steady state response, of equation (17) under harmonic force F(t) of frequency 'ω' is given by:

$$\xi_r(t) = \frac{\varphi_r{}^T F(t)}{j\omega\text{-}\lambda_r} \tag{18}$$

Where $'j'$ is the imaginary unit, i.e. $j = \sqrt{-1}$.

Substituting equation (18) into equation (14) gives

$$X(t) = \sum_{r=1}^{8(N+1)} \frac{\Psi_r \varphi_r{}^T}{j\omega\text{-}\lambda_r} F(t) \tag{19}$$

Therefore, Frequency Response Function matrix (H) in state space may be written as:

$$\underline{H} = \sum_{r=1}^{8(N+1)} \frac{\psi_r \varphi_r{}^T}{j\omega\text{-}\lambda_r} = \sum_{r=1}^{4(N+1)} \frac{\psi_r \varphi_r{}^T}{j\omega\text{-}\lambda_r} + \sum_{r=1}^{4(N+1)} \frac{\bar{\psi}_r \bar{\varphi}_r{}^T}{j\omega\text{-}\bar{\lambda}_r} \tag{20}$$

Where over bar $(\bar{\ })$ represents complex conjugate of a vector or scalar.

By using the relationship of $\psi$ and $\varphi$ with $u$ and $v$ from equation (10), the Frequency Response Function matrix $(H)$ relating generalized displacement may be written as:

$$\underline{H} = \sum_{r=1}^{8(N+1)} \frac{\begin{Bmatrix} \lambda_r u_r \\ u_r \end{Bmatrix} \begin{Bmatrix} \lambda_r v_r \\ v_r \end{Bmatrix}^T}{j\omega - \lambda_r} = \sum_{r=1}^{8(N+1)} \frac{\begin{pmatrix} \lambda_r{}^2 u_r v_r{}^T & \lambda_r{}^2 u_r v_r{}^T \\ \lambda_r{}^2 u_r v_r{}^T & \lambda_r{}^2 u_r v_r{}^T \end{pmatrix}}{j\omega\text{-}\lambda_r} \tag{21}$$

Thus Frequency-displacement response functions (H), further referred to as Frequency ResponseFunctions may be expressed as:

$$\underline{H} = \begin{bmatrix} H_{yy} & H_{yz} \\ H_{zy} & H_{zz} \end{bmatrix} \quad (22)$$

After following Lee [3], the directional Frequency Response Function (dFRF) and reverse directional Frequency Response Function(r-FRF) can be written as:

$$\begin{aligned} 2H_{pg} &= H_{yy} + H_{zz} - i(H_{yz} - H_{zy}); \\ 2H_{p\hat{g}} &= H_{yy} - H_{zz} + i(H_{yz} + H_{zy}); \\ 2H_{\hat{p}g} &= H_{yy} - H_{zz} - i(H_{yz} + H_{zy}); \\ 2H_{\hat{p}\hat{g}} &= H_{yy} + H_{zz} + i(H_{yz} - H_{zy}); \end{aligned} \quad (23)$$

Here $H_{pg}$ and $H_{\hat{p}\hat{g}}$ are directional Frequency Response Function(dFRF)matrix and $H_{p\hat{g}}$, $H_{\hat{p}g}$ are reverse directional Frequency Response Function(r-FRF)matrix (Lee[3]).Where p and g represent the complex linear displacement and complex force at any particular node and are given by equation:

$$p(t) = y(t) + jz(t) \text{ and } g(t) = f_y(t) + jf_z(t) \quad (24)$$

$\hat{p}$ and $\hat{g}$ represent the complex conjugate of p and g respectively. Therefore $H_{pg}$ ($H_{\hat{p}\hat{g}}$)is defined as the frequency response of the rotor in the forward(backward)direction to rotating excitation in the forward (backward)direction, and is termed as normal dFRF, whereas $H_{p\hat{g}}$ ( $H_{\hat{p}g}$ )represents the frequency response of the rotor in the forward(backward)direction to rotating excitation in the backward(forward)direction and is termed as the reverse dFRF(Kessler and Kim[14].Following Lee we can write

$$H_{\hat{p}g}(j\omega) = \overline{H}_{p\hat{g}}(-j\omega) \text{ and } H_{pg}(j\omega) = \overline{H}_{\hat{p}\hat{g}}(-j\omega) \quad (25)$$

Using the Complex Response Functions the anisotropy/asymmetry and cracks in rotating machinery can be detected.

# CHAPTER THREE

## Results and Discussion

This chapter includes the various numerical results based on the modal characteristic of a simply supported viscoelastic rotor shaft system. The different modal study comprises the decay rate plot, modal damping ratio, mode shape and DFRF. A finite element code is written in MATLAB for numerical simulation of the theoretical work.

## 3.1 THE ROTOR-SHAFT SYSTEM

The schematic diagram for one such shaft system is shown in figure 3.1. A rotating shaft system having three discs, with simply supported ends is considered. Table 1 shows the dimension and material properties of the steel rotor. The dimensions of these three discs are shown in Table 2.

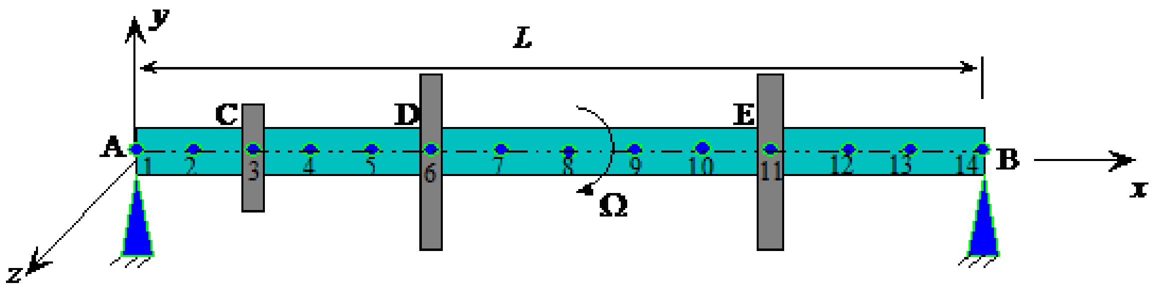



**Figure 3.1 Schematic Diagram of the Rotor**

| Material | Density ($kg/m^3$) | Young's Modulus (GPa) | Length(m) | Diameter(m) | Damping Coefficient (N-s/m) |
|---|---|---|---|---|---|
| Mild Steel | 7800 | 200 | 1.3 | 0.2 | 0.0002 |

**Table 1 Rotor Material and its Properties**

| Disc | Diameter(m) | Thickness(m) |
|---|---|---|

| | | |
|---|---|---|
| 1 | 0.24 | 0.05 |
| 2 | 0.40 | 0.05 |
| 3 | 0.40 | 0.06 |

**Table 2 Disc parameters for 3 disc rotor**

## 3.2 STABILITY LIMIT OF SPIN SPEED:

### 3.2.1. DECAY RATE PLOT

Figure 3.2shows the decay rate plot (i.e. the variation of maximum real part with the spin speed) of two consecutive modes. After a certain speed the maximum real part line cuts the zero line, the system becomes unstable and corresponding speed is called stability limit of spin speed (SLS). From the graph SLS is 4353rpm. First mode cuts the zero line before the second mode, this implies that the fist mode becomes unstable .If real part is negative, the amplitude decays in time then the rotor has a stable behavior because the whirl motion tends to reduce its amplitude. If real part is positive, the amplitude grows exponentially, the motion is unstable, as any small perturbation can trigger this self-excited whirling.

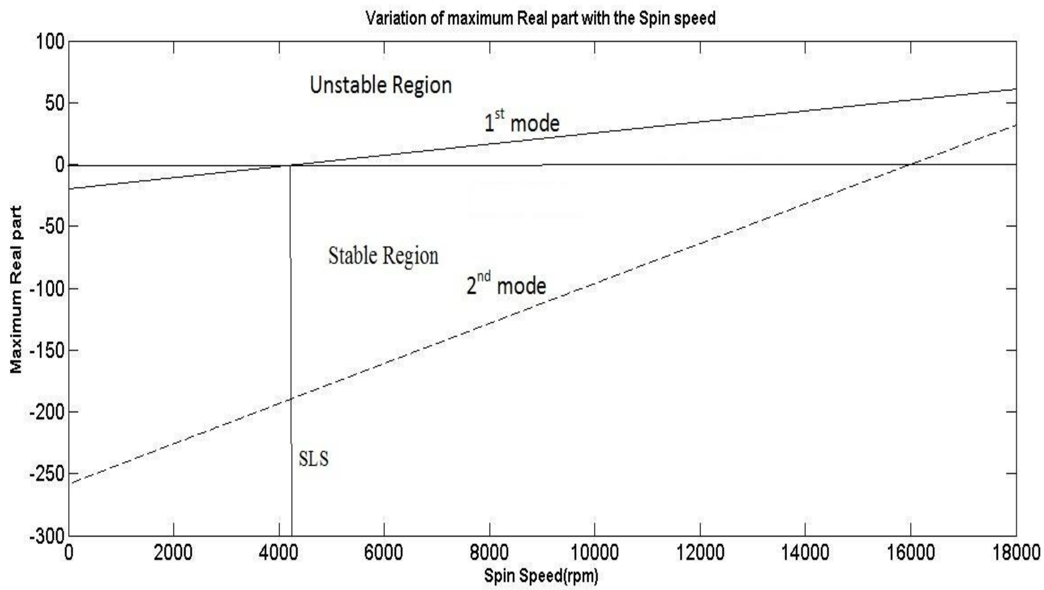



**Figure 3.2 Variation of Maximum real Part with the spin speed**

### 3.2.2. MODAL DAMPING FACTOR

Figure 3.3 shows the variation of modal damping factor with the rotor spin speed. In case of 1F and 2F modes, the modal Damping factor decreases as the spin speed increases. Whereas the modal damping factor increases in case of 1B and 2B mode.Positive modal damping factor indicates stability as vibratory energy is dissipated and negative modal damping signifies instability as rotary energy supports rotor whirl by adding energy.This shows that forward mode tend to destabilize due to internal material damping and backward mode does not have any effect on instability.Like decay rate plot, the SLS can be obtained from this plot, till which none of the modal damping factors is negative. From the graph, it can be seen that the 1F mode becomes unstable at the 1st critical speed (Ωcr) of 4349 rpm.

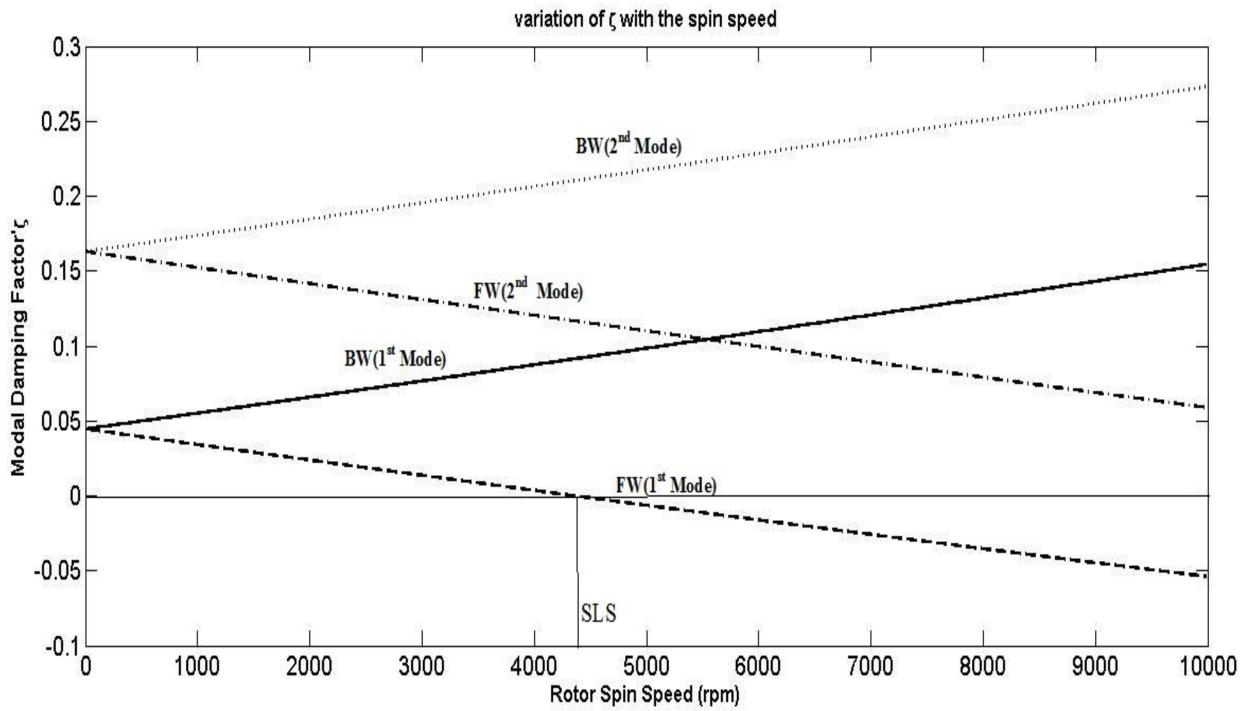



**Figure 3.3 Variation of modal damping factor with the spin speed**

## 3.3 CAMPBELL DIAGRAM

Figure 3.4 Shows the Campbell diagram of the rotor-shaft system, when the shaftsinternal material damping is considered. The graph is plotted by using the whirl frequencies (obtained from the imaginary part of the eigenvalues). There are 2 forward-whirling modes, marked sequentially in the ascending order of frequency, by ‘FW(1$^{st}$ mode) and FW(2$^{nd}$ mode) in which the rotor whirls in the direction of the spin and there are two backward whirling mode ‘BW(1$^{st}$ mode)and BW(2$^{nd}$ mode), in which the rotor whirls opposite to the direction of spin. The Synchronous Whirl Line is marked as ‘SWL’. The speed corresponding to the first point of intersection of ‘SWL’ with the Campbell diagram is the critical speed shown as ‘Ωcr’ in Figure 3.4. For this example Ωcr(1$^{st}$ FW)=4348 rpm and Ωcr(2$^{nd}$ FW)=15990 rpm .Hence the resonance of the system can be found out using the Campbell diagram.

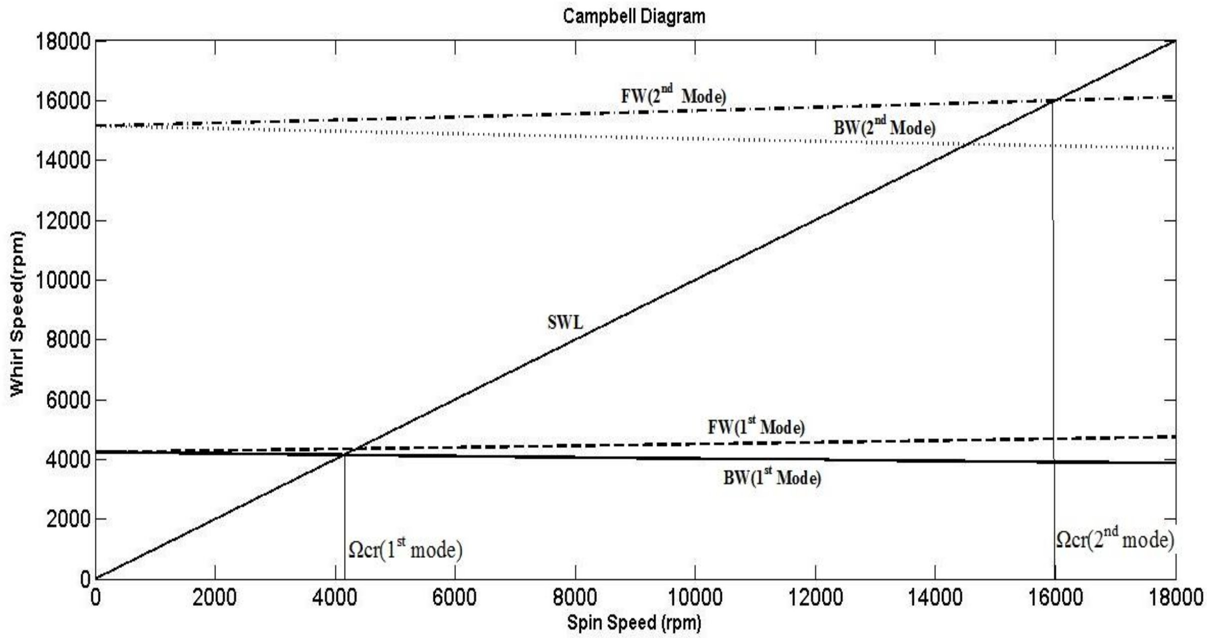
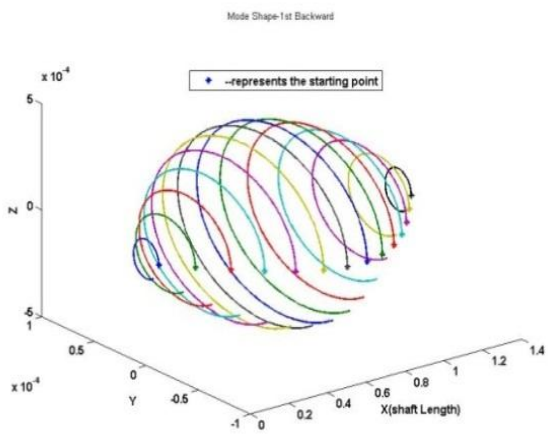



**Figure 3.4Campbell Diagram**

## 3.4 THREE DIMENSIONAL MODE SHAPES OF SIMPLY SUPPORTED ROTOR

The mode shapes of a rotating shaft indicate the locus of any point of the shaft during whirling motion. First backward and forward Mode shapes of the simply supported rotor are plotted using the eigenvectors. Figure 3.5 and 3.6 shows the first three dimensional mode shape for undamped rotor $(\eta_v = 0)$ and the figure 3.7 and 3.8 show the same plot for damped rotor (considering internal damping i.e. ($\eta_v \neq 0$). '*' indicates the starting point of the whirl. In the backward mode the whirl lines rotate clockwise direction whereas forward whirl lines rotate counter clockwise direction. The mode shape is symmetric about *y*-axis and *z*-axis for undamped rotor, but it is not maintained for damped rotor. It is due to the incorporation of skew symmetric circulatory matrix in the system equation.



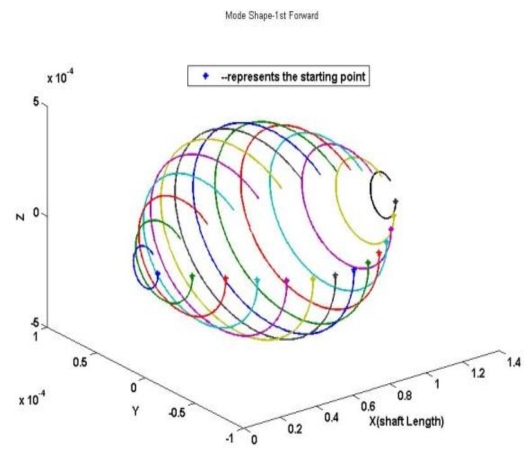
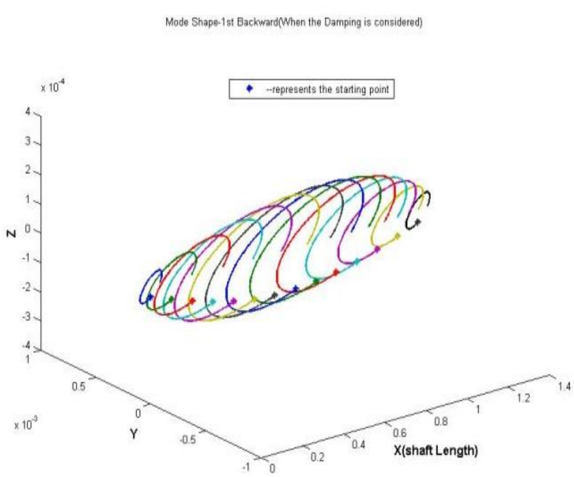




**Figure 3.5 1st Backward ($\eta_v$=0)** **Figure 3.61st Forward ($\eta_v$=0)**



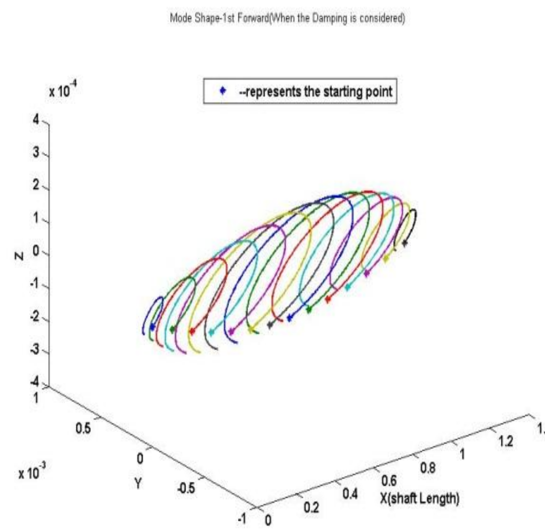




**Figure 3.71st Backward** ($\eta_v \neq 0$) **Figure 3.8 1st Forward** ($\eta_v \neq 0$)

## 3.5DIRECTIONAL FREQUENCY RESPONSE FUNCTION

Figure 3.9 shows the plot of dFRF ( $H_{pg}$ ) at node 6 of the rotor Shaft system.In this plot backward modes appear in the negative frequency region and forward modes appear in the positive frequency region. Therefore we noted that the dFRF plot has the ability to separate the forward and backward modes, while in FRF plot these modes are mixed, resulting in more difficulty to the process of parameter estimation. The dFRF has the advantage of separating the modes in the negative and positive excitation frequency region, which otherwise cannot be identified through classical FRF due to its conjugate even property.

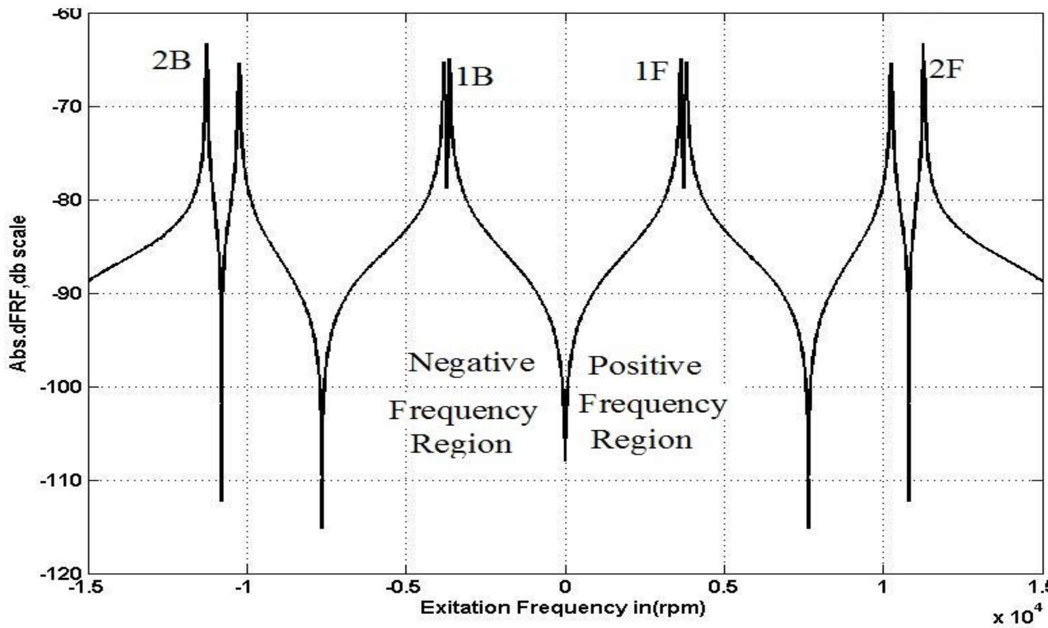



**Figure 3.9dFRF ( $H_{pg}$ ) plot, at node 6,( $K_{yy} = 7e7, K_{zz} = 5e7, c_{yy} = 0, c_{zz} = 0, \eta_v = 0$ )**

# CHAPTER FOUR

## Conclusionsand scope for future work

## 4.1 CONCLUSIONS

This work investigates the modal analysis of a rotor-shaft system with simply supported ends, by considering the internal material damping of the Rotor. For this the finite element method is used for modelling the rotor shaft. The following important conclusions are obtained at the end.

1. During the forward whirl, damping decreases, as the spin speed increases and in backward whirl damping increases, as the spin speed increases. Positive value of damping factor indicates the stability of the system. Using this, the stability of the system is found out.
2. The critical speed during the forward whirl and backward whirl are found out using a Campbell diagram. The speed corresponding to the first point of intersection of 'SWL' with the Campbell diagram is the critical speed. For the stability of the system, the system must be operated at a speed less than the critical speed.
3. The mode shapes are plotted using the eigenvectors. The mode shapes are symmetric for undamped rotor where as those are Unsymmetric for damped rotor.
4. The study also included the effect of shaft material damping on directional frequency response characteristics. As rotor spin speed increases, the contribution of forward modes in the directional frequency response increases and that of backward modes decreases. This observation can also be used as an indicator for the presence of shaft material damping.

Hence the Dynamic behaviour of the system is identified by performing themodal analysis and which helps in the dynamic design of rotors. It can be concluded that the complex modal analysis is a tool to get an idea about the dynamic behaviour of the system.

## 4.2 SCOPE FOR FUTURE WORK:

This study has given birth to different other possibilities which may be taken up as future research activities in this area.

1. Sometimes the equations of motion of rotating shaft consist of higher order model. The number of order depends on the nature of material. The study of complex modal behaviour of a second order system is little straight forward. The present procedure can easily be extended for obtaining the dynamic behaviour of viscoelastic rotor having a higher order system.
2. Obtaining the modes for the higher order system is cumbersome and complicated as compare to general second order system. Because the eigen vector comprises the displacement function as well as their higher order derivatives. Thus the present complex modal analysis technique can be used to get the equivalent reduced form of the higher order model.
3. Using the Complex Frequency Response Functions the anisotropy, asymmetry and cracks in rotating machinery can be detected. The complex modal analysis of complete bladed disc will also help to detect the instant failure of any disc during operation.
4. The modal analysis of non axy symmetric rotor is not available in the literature. To get an insight idea of the dynamic characteristics of the non axy - symmetric rotor the complex modal analysis is very important.

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- ✓ **P. Mutalikdesai**, S. Chandraker, H. Roy, 2013, "Modal Analysis of Damped Rotor using Finite Element Method" Proceedings of (AMAAS), March 01-02, NIT Rourkela, India.
- ✓ S. Chandraker, **P. Mutalikdesai**, H. Roy, 2013, "Complex modal analysis of damped rotor using finite element method" Proceedings of (ICAME), May 29-31, COEP Pune, Maharashtra, India, Paper ID: S16/P4.